

Relativity tests and their motivation

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Abstract. Some motivations for Lorentz-symmetry tests in the context of quantum-gravity phenomenology are reiterated. The description of the emergent low-energy effects with the Standard-Model Extension (SME) is reviewed. The possibility of constraining such effects with dispersion-relation analyses of collider data is established.

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INTRODUCTION

One of the principal cornerstones of present-day physics is the special theory of relativity. It was established at the beginning of the last century, and it has significantly transformed our understanding of space and time. Despite substantial experimental scrutiny, there exists no credible observational evidence for deviations from relativity theory. As a matter of fact, Lorentz invariance, the symmetry that underlies special (and general) relativity, has acquired a venerable status. For example, it is an ingredient in most theoretical approaches to physics beyond the Standard Model and general relativity.

Nevertheless, the last decade has witnessed a revival of interest in experimental tests of Lorentz symmetry. This renewed interest stems primarily from the realization that a more complete theory unifying quantum physics and gravity is likely to affect the structure of spacetime at small distance scales. In fact, the majority of theoretical approaches to quantum gravity (although based on Lorentz symmetry) can accommodate minuscule deviations from special relativity in the ground state. Such mechanisms for Lorentz violation exist, for example, in string theory, spacetime-foam models, non-commutative field theory, and cosmologically varying scalars [1].

To identify and analyze present and near-future experimental tests of these ideas, a general framework for the description of Lorentz violation at currently attainable energies is needed. Such a framework, known as the Standard-Model Extension (SME), has been developed in a series of papers [2]. The SME is an effective field theory that incorporates practically all established physics in the form of the Standard-Model and general-relativity Lagrangians. In addition, it contains Lorentz- and CPT-breaking contributions formed by contracting external, non-dynamical vectors and tensors with conventional particle and gravitational fields to form coordinate scalars. The prescribed background vectors and tensors control the size and type of Lorentz and CPT violation and are amenable to experimental searches. The dominant Lorentz- and CPT-breaking effects are expected to arise from the power-counting renormalizable contributions to the SME. This particular subset of the SME is called the minimal Standard-Model Extension

(mSME); in the past, it has provided the basis for numerous experimental [3, 4] and theoretical [5] investigations of relativity theory.

One prediction of the SME that has been particularly popular is the modification of one-particle dispersion relations. The novel correction terms in such dispersion relations typically involve the Lorentz-violating background contracted with certain powers of the particle’s momentum, so that the Lorentz-breaking effects tend to be more significant at higher energies. Therefore, ultrahigh-energy cosmic rays (UHECRs) have traditionally been used for kinematical dispersion-relation tests of Lorentz symmetry [6]. However, the kinematics of an UHECR collision involves the (modified) dispersion relations of *all* involved particles including the primary, but the nature of the primary is often difficult to establish. For this reason, it is interesting to consider also Lorentz tests via particle collisions in a controlled laboratory environment at the cost of being confined to a lower-energy regime. In what follows, we will therefore focus on dispersion-relation tests of special relativity at highest-energy particle colliders [7].

It turns out that Lorentz tests at particle accelerators are particularly sensitive to the electron–photon sector of the mSME. Gravitational physics can be safely ignored. The mSME coefficients applicable in this context are $(k_F)^{\mu\nu\rho\lambda}$, $(k_{AF})^\mu$, b^μ , $c^{\mu\nu}$, $d^{\mu\nu}$, and $H^{\mu\nu}$; all of these non-dynamical background vectors and tensors are spacetime constant. It is important to note that the coefficients $c^{\mu\nu}$ and $\tilde{k}^{\mu\nu} \equiv (k_F)_\alpha^{\mu\alpha\nu}$ are physically equivalent in an electron–photon system. This equivalence arises because suitable coordinate rescalings freely transform the $\tilde{k}^{\mu\nu}$ and $c^{\mu\nu}$ parameters into one another [8, 9]. From a physics perspective, this represents the fact that we may choose to measure distances with a ruler composed of electrons ($c^{\mu\nu} = 0$), or with a ruler composed of photons ($\tilde{k}^{\mu\nu} = 0$), or any other ruler ($c^{\mu\nu}, \tilde{k}^{\mu\nu} \neq 0$). We exploit this freedom by selecting the specific scaling $c^{\mu\nu} = 0$ (corresponding to an “electron ruler”) in intermediate calculations. However, we state the final result in a scaling-independent (i.e., “ruler-independent”) way and reinstate the $c^{\mu\nu}$ coefficient for generality.

In principle, all of the above coefficients can contribute to the kinematics of the electron–photon vertex. However, prior experimental bounds on Lorentz violation establish that the dominant one of the above mSME coefficient is k_F [4]. This coefficient causes a direction- and polarization-dependent speed of light [8]. A number of its components have been tightly constrained with astrophysical polarimetry [10], Michelson–Morley tests [4, 11], and Compton scattering [12]. We will place limits on the $\tilde{\kappa}_{\text{tr}}$ piece of $\tilde{k}^{\mu\nu}$, which is its isotropic component [8]. At the time of the analysis, it obeyed the weakest limits, so all other components of $\tilde{k}^{\mu\nu}$ components can also be set to zero in this context. An mSME calculation then shows that the photon’s dispersion relation is modified: in the presence of $\tilde{\kappa}_{\text{tr}}$, it is given by [8]

$$E_\gamma^2 - (1 - \tilde{\kappa}_{\text{tr}})\vec{p}^2 = 0. \quad (1)$$

Here, $p^\mu \equiv (E_\gamma, \vec{p})$ is the photon’s 4-momentum, and Eq. (1) holds at leading order in $\tilde{\kappa}_{\text{tr}}$. This dispersion relation can be interpreted as a nontrivial isotropic refractive index n of the vacuum:

$$n = 1 + \tilde{\kappa}_{\text{tr}} + \mathcal{O}(\tilde{\kappa}_{\text{tr}}^2). \quad (2)$$

Note in particular that the physical speed of light is $(1 - \tilde{\kappa}_{\text{tr}})$ (i.e., different from the usual $c = 1$). We also remark that the electron’s dispersion relation $E(p) = \sqrt{m_e^2 + p^2}$

remains unaltered with our choice of coordinate scaling. In what follows, we treat the two cases $\tilde{\kappa}_{\text{tr}} < 0$ and $\tilde{\kappa}_{\text{tr}} > 0$ separately because they lead to different phenomenological effects.

PHOTON DECAY

For negative $\tilde{\kappa}_{\text{tr}} < 0$, photons travel faster than the maximal attainable speed (MAS) of electrons. This introduces photon instability: for photon energies E_γ above the threshold

$$E_{\text{pair}} = \frac{2m_e}{\sqrt{\tilde{\kappa}_{\text{tr}}(\tilde{\kappa}_{\text{tr}} - 2)}} = \sqrt{\frac{2}{-\tilde{\kappa}_{\text{tr}}}} m_e + \mathcal{O}\left(\sqrt{\tilde{\kappa}_{\text{tr}}}\right), \quad (3)$$

photon decay into an electron–positron pair is kinematically allowed [13, 9]. This threshold condition can be established with the aid of the modified dispersion relation (1). The leading-order decay rate of this process is given by [13, 9]

$$\Gamma_{\text{pair}} = \frac{2}{3} \alpha E_\gamma \frac{m_e^2}{E_{\text{pair}}^2} \sqrt{1 - \frac{E_{\text{pair}}^2}{E_\gamma^2}} \left(2 + \frac{E_{\text{pair}}^2}{E_\gamma^2}\right), \quad (4)$$

where $\alpha \simeq \frac{1}{137}$ denotes the fine-structure constant. Note that this process is highly efficient. For example, a 40 GeV photon with energy 1% above threshold would decay after traveling about $30 \mu\text{m}$.

The absence of such a photon-decay effect in nature can be used to obtain limits on negative values of $\tilde{\kappa}_{\text{tr}}$ as follows. Suppose long-lived photons with a known energy E_γ are observed to exist. Such photons must essentially be below threshold $E_\gamma < E_{\text{pair}}$, for otherwise they would decay rapidly according to Eq. (4). Using the sub-threshold condition $E_\gamma < E_{\text{pair}}$ in Eq. (3) yields

$$E_\gamma \lesssim \sqrt{\frac{2}{-\tilde{\kappa}_{\text{tr}}}} m_e \quad \text{or equivalently} \quad \tilde{\kappa}_{\text{tr}} \gtrsim -2 \frac{m_e^2}{E_\gamma^2}. \quad (5)$$

It follows that stable photons with higher energies E_γ give stronger bounds on negative values of $\tilde{\kappa}_{\text{tr}}$.

Hadron colliders generate the highest-energy photons and therefore give tight Earth-based experimental limits on negative $\tilde{\kappa}_{\text{tr}}$. This leads us to consider Fermilab’s Tevatron $p\bar{p}$ collider with center-of-mass energies up to 1.96 TeV. At the Tevatron, isolated-photon production with an associated jet has been investigated with the D0 detector because of its importance for QCD studies. In this context, the photon-energy bin at the ultraviolet end of the recorded spectrum extended from 300 GeV to 400 GeV. We can therefore conservatively take $E_{\text{pair}} > E_\gamma \simeq 300 \text{ GeV}$. With Eq. (5), we then arrive at the constraint

$$-5.8 \times 10^{-12} \lesssim \tilde{\kappa}_{\text{tr}} - \frac{4}{3} c^{00}, \quad (6)$$

where we have reinstated the contribution of the electron’s c^{00} coefficient.

VACUUM CHERENKOV RADIATION

For positive $\tilde{\kappa}_{\text{tr}} > 0$, photons are stable. The speed of light is now $(1 - \tilde{\kappa}_{\text{tr}})$. Note in particular that this speed is slower than the MAS of the electrons. In analogy to conventional electrodynamics inside a macroscopic medium, this suggests a Cherenkov-type effect [14]: charges moving faster than the modified speed of light $(1 - \tilde{\kappa}_{\text{tr}})$ become unstable against the emission of photons. Employing the Lorentz-violating dispersion relation (1), one can indeed establish that electrons with energies E above the threshold

$$E_{\text{VCR}} = \frac{1 - \tilde{\kappa}_{\text{tr}}}{\sqrt{(2 - \tilde{\kappa}_{\text{tr}})\tilde{\kappa}_{\text{tr}}}} m_e = \frac{m_e}{\sqrt{2\tilde{\kappa}_{\text{tr}}}} + \mathcal{O}(\sqrt{\tilde{\kappa}_{\text{tr}}}) \quad (7)$$

emit Cherenkov radiation. We remark that the threshold (7) can alternatively be derived from the usual Cherenkov condition that the electron must be faster than the speed of light $(1 - \tilde{\kappa}_{\text{tr}})$.

Paralleling the photon-decay case in the previous section, we want to determine an experimental limit on $\tilde{\kappa}_{\text{tr}}$ through the non-observation of vacuum Cherenkov radiation. To this end, we need to establish that this effect would be efficient enough for a rapid deceleration of charges with $E > E_{\text{VCR}}$ to energies below threshold. One can show that near E_{VCR} , the dominant deceleration process is single-photon emission with an estimated rate of [15]

$$\Gamma_{\text{VCR}} = \alpha m_e^2 \frac{(E - E_{\text{VCR}})^2}{2E^3}, \quad (8)$$

where α is again the fine-structure constant, and E denotes the electron energy, as before. A numerical evaluation of this expression indeed shows that the emission process is quite efficient, and above-threshold electrons would be extremely short-lived.

We can now employ the threshold condition (7) together with the existence of high-energy electrons to place a limit on positive values of $\tilde{\kappa}_{\text{tr}}$. The observation of long-lived electrons at a known energy E practically implies $E_{\text{VCR}} > E$. Using this information in Eq. (7) gives

$$E \lesssim \frac{m_e}{\sqrt{2\tilde{\kappa}_{\text{tr}}}} \quad \text{or equivalently} \quad \tilde{\kappa}_{\text{tr}} \lesssim \frac{1}{2} \frac{m_e^2}{E^2}. \quad (9)$$

It is apparent that the limit on positive $\tilde{\kappa}_{\text{tr}}$ gets tighter with higher energies E of the long-lived electrons.

The highest laboratory-frame electron energy at an Earth-based collider was reached at LEP, where the value $E_{\text{LEP}} = 104.5 \text{ GeV}$ was attained. Employing Eq. (8), we can establish that if $E_{\text{VCR}} = 104 \text{ GeV}$, electrons initially accelerated to 104.5 GeV would be rapidly decelerated by the emission of Cherenkov radiation to an energy below E_{VCR} over a $1/e$ length of roughly 95 cm . The total energy loss due to the Cherenkov effect in such a scenario would far exceed the value allowed by measurements. With Eq. (7) at hand, the requirement that E_{VCR} be greater than 104 GeV yields

$$\tilde{\kappa}_{\text{tr}} - \frac{4}{3} c^{00} \leq 1.2 \times 10^{-11}, \quad (10)$$

where we have again included the dependence on c^{00} for generality.

CONCLUSIONS

We have shown that data from highest-energy particle colliders can be used to extract competitive limits on isotropic Lorentz violation in the electron–photon system. Combining the results (6) and (10), we obtain the two-sided limit

$$-5.8 \times 10^{-12} \leq \tilde{\kappa}_{\text{tr}} - \frac{4}{3} c^{00} \leq 1.2 \times 10^{-11} . \quad (11)$$

We remark that other aspects of collider physics (namely modifications of synchrotron radiation) yield further improvements of this limit at the 10^{-15} level [16].

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